Chapter R.4 Factoring Polynomials

• Introduction to Factoring

To **factor** an expression means to write the expression as a product of two or more factors.

Sample Problem: Factor each expression. a. 15 b. x^2+3x+2

Solution: a. 15 = $5 \bullet 3$ b. $x^2+3x+2 = (x+2)(x+1)$

Note that 5 and 3 are called factors of 15, while (x + 2) and (x + 1) are called factors of x^2+3x+2.

● FACTORING OUT GCF FROM POLYNOMIALS

The **GCF (Greatest Common Factor)** is the product of the largest factor of the numerical coefficients and, if all terms have a common variable, the smallest degree of the variable. To factor polynomials, we first look for a **GCF** in all terms and use the distributive property in reverse.

Sample Problem: Factor out the GCF. $24x^5-30x^3$

Solution: The GCF of 24 and 30 is 6. Since both terms have an *x*, then the GCF between x^5 and x^3 is x^3 *(smallest degree)*. So the GCF is $6x^3$. We can now write the expression as follows:

$24x^5-30x^3 = 6x^3 \bullet 4x^2 - 6x^3 \bullet 5 = 6x^3(4x^2-5)$ ***Check:*** $6x^3(4x^2-5) = 24x^5-30x^3$

Student Practice: Factor out the GCF of each polynomial using the distributive property. Check your answer by multiplying.

1. $15x+25$

2. $12y-4$

3. $50x^2-10x$

4. x^3+9x^2

5. $9y^2 + 6y$

6. $4x^4 - 8x^3$

7. $3x^4 - 6x^3 + 9x^2$

8. $5x^3y - 15x^2y + 10xy$

9. $x(x-1) + 2(x-1)$

10. $5x(x+7) - 2(x+7)$

11. $x^2(a+b) + (a+b)$

● FACTORING POLYNOMIALS BY GROUPING (4 TERMS)

To factor polynomials by grouping, (useful when polynomials have 4 terms)

1. First check for a GCF in all 4 terms.
2. Rewrite the first two terms by factoring out the GCF.
3. Bring down the middle operation, then rewrite the second pair by factoring out the GCF.
4. Factor out the common factor.

Sample Problem: ***Factor by grouping.*** $x^3 + 6x^2 - 2x - 12$

Solution: $x^3 + 6x^2 - 2x - 12$

$x^2(x+6) - 2(x+6)$

$(x^2 - 2)(x+6)$

Check: $(x^2 - 2)(x+6) = x^3 + 6x^2 - 2x - 12$

Student Practice: Factor each polynomial by grouping.

1. $x^3 + 7x^2 + 3x + 21$
2. $x^3 - 5x^2 + 7x - 35$
3. $4x^3 - 8x^2 - 9x + 18$
4. $4x^3 - 6x^2 - 10x + 15$
5. $x^4 - 2x^3 + x - 2$
6. $2x^3 - 5x^2 - 2x + 5$

Factoring Trinomials $x^2 + bx + c$

• FACTORING TRINOMIALS (3 TERMS)

To better understand how to factor trinomials, let's recall how to use the FOIL method.

Sample Problem: ***Multiply.*** $(x+7)(x-5)$

Solution: $(x+7)(x-5) = x^2 - 5x + 7x - 35 = x^2 + 2x - 35$

To factor trinomials of the form $x^2 + bx + c$**,**

1. Write two sets of parenthesis as such: $x^2 + bx + c = (x \quad\quad)(x \quad\quad)$
2. Place an *x* as the first term in each binomial.
3. Find two factors of *c* that sum up to the middle coefficient *b*. Write these factors as the second term of each binomial.
4. **CHECK YOUR ANSWER BY FOILING**.

It is important to note that there are two types of trinomials; those with a coefficient of 1 for the x^2, such as $x^2 + 5x + 4$; and those with a coeffiecient of something else, such as $3x^2 + 5x - 2$. Those with a coefficient of 1 for the x^2 are easier to factor and can be done as follows.

Sample Problem 1: ***Factor.*** $x^2 - 10x - 24$

Solution: $x^2 - 10x - 24 = (x \quad\quad)(x \quad\quad)$

We know the first two terms must be *x*. To find what numbers to use, we list all numbers that multiply to –24 and find the combination that adds to –10.

–24	
1	24
2	**–12**
3	8
4	6

This sum will be –10.

Will only add to –10 if we use –4 and –6, but product will then be +24.

$x^2 - 10x - 24 = (x + 2)(x - 12)$

Check: $(x-12)(x+2) = x^2 + 2x - 12x - 24$
$= x^2 - 10x - 24$

Sample Problem 2: ***Factor.*** $x^2 - 9x + 18$

Solution: $x^2 - 9x + 18 = (x \quad)(x \quad)$

We know the first two terms must be *x*. To find what numbers to use, we list all numbers that multiply to + 18 and find the combination that adds to –9.

+18	
1	18
2	9
–3	**–6**

This sum will be -9.

$x^2 - 9x + 18 = (x-3)(x-6)$

Check: $(x-3)(x-6) = x^2 - 6x - 3x + 18$
$= x^2 - 9x + 18$

Note: The order in which the factors are written is not important. What IS important is the sign used with each factor.

Student Practice: Factor each trinomial.

1. $x^2 + 8x + 12$

2. $x^2 - 3x - 18$

3. $x^2 + 25x + 24$

4. $x^2 - 13x + 36$

5. $y^2 - y - 72$

6. $x^3 + x^2 - 12x$

7. $3x^2 - 15x + 18$

8. $36x - 18x^2 + 2x^3$

9. $x^2 - 7x - 10$

10. $x^2 + 13x - 15$

11. $x^4 - 10x^2 + 25$

12. $x^2 + 10xy + 16y^2$

Factoring More Trinomials $ax^2 + bx + c$

To factor trinomials of the form $ax^2 + bx + c$,

1. Multiply the coefficients *a* and *c.*
2. Find two factors of the product *ac* that sum up to the middle coefficient *b*.
3. Rewrite the trinomial as a polynomial with four terms by splitting the middle term using the factors found in step 2.
4. Factor the polynomial by grouping.
4. **CHECK YOUR ANSWER BY FOILING**.

Sample Problem 1: ***Factor.*** $5x^2 + 17x - 12$

Grouping approach: In the grouping approach, we combine what we learned before. We first start by multiplying the first and last coeffiecients; in this case $5 \bullet -12 = -60.$
We then find factors of –60 that sum up to +17.

–60	
1	60
2	30
–3	**20**
4	15
5	12
6	10

These numbers will add to +17.

We now change the trinomial into a polynomial with 4 terms by splitting the middle term using the factors we just found. We then use factoring by grouping to solve.

$$\begin{aligned} 5x^2 + 17x - 12 &= 5x^2 - 3x + 20x - 12 \\ &= x(5x-3) + 4(5x-3) \\ &= (5x-3)(x+4) \end{aligned}$$

Sample Problem 2: ***Factor.*** $10x^2 + 39x + 14$

Grouping approach:
We first start by multiplying the first and last coeffiecients; in this case $10 \bullet 14 = 140.$
We then find factors of 140 that sum up to +39.

140	
1	140
2	70
4	**35**
5	28
7	20
10	14

These factors add up to 39.

We now change the trinomial into a polynomial with 4 terms by splitting the middle term using the factors we just found. We then use factoring by grouping to solve.

$$\begin{aligned}10x^2+39x+14 &= 10x^2+4x+35x+14\\ &= 2x(5x+2)+7(5x+2)\\ &= \boxed{(5x+2)(2x+7)}\end{aligned}$$

OPTIONAL: ALTERNATIVE METHOD OF FACTORING USING F O I L

Sample Problem 3: ***Factor.*** $5x^2+17x-12$

Solution:

$5x^2+17x-12 = (5x \quad\quad 3\ )(x \quad\quad 4\ )$

$5x^2 \quad 12$

$5x^2+17x-12 = (5x \quad\quad 3\ )(x \quad\quad 4\ )$

$3x$

$20x$

$5x^2 \quad 12$

$5x^2+17x-12 = (5x \quad\quad 3\ )(x \quad\quad 4\ )$

$-3x$

$+20x$

$+17x$

So, $5x^2+17x-12 = \boxed{(5x-3)(x+4)}$ ***Check:*** $(5x-3)(x+4) = 5x^2+20x-3x-12$
$= 5x^2+17x-12$

***Note:* Whenever the coefficient of x^2 is 1, the factors chosen for the last <u>term can be placed in any order</u> and must add to the middle term. However, if the coefficient of x^2 is not 1, then the factors chosen for the last term <u>must be placed in the proper order</u> and the inner/outer products must sum up to the middle term.**

Try: $7x^2+31x-20$

Student Practice: Factor each trinomial.

1. $2x^2 - 7x + 5$

2. $3x^2 + 7x - 6$

3. $6x^2 + 5x - 6$

4. $8x^2 - 18x - 5$

5. $4x^5 - x^4 - 5x^3$

6. $3x^2 - 26x + 16$

7. $2x^4 - 7x^2 - 15$

Factoring Difference of Two Squares (DOTS)

● FACTORING BINOMIALS (2 TERMS)

Notice what happens when we multiply two binomials with the same two terms, only one with addition and one with subtraction.

$(x+7)(x-7) = x^2 - 7x + 7x - 49 = x^2 - 49$

Tip: You can also view binomials as trinomials with a 0 middle term, such as $x^2 + 0x - 49$***, and then factor as a trinomial finding factors of -49 that add to 0.***

The result is a binomial that is a **difference of two perfect squares**. In other words,

$x^2 - 49 = (x)^2 - (7)^2$

Any binomial that is a **difference** of **two perfect squares** can be factored using **DIFFERENCE OF TWO SQUARES**.

$$a^2 - b^2 = (a+b)(a-b)$$

Sample Problem: ***Factor.*** $x^2 - 9$

Solution: $x^2 - 9 = (x)^2 - (3)^2 = (x+3)(x-3)$

Note: Tthe two terms in the binomial <u>MUST</u> be perfect squares and there <u>MUST</u> be a difference (subtraction).

Student Practice: Factor each binomial. Remember to first check for a GCF.

1. $x^2 - 25$

2. $y^2 - 36$

3. $27a^3 - 3a$

4. $2x^2 - 32y^2$

5. $w^2 + 16$

6. $x^4 - 81$

7. $4x^3 - 9x$

8. $x^2 - 12$