REVIEW ON MULTIPLYING POLYNOMIALS

To multiply two polynomials, simply multiply each term of one polynomial by each term of the other polynomial. **Remember to add exponents when multiplying like bases and to combine like terms if possible.**

Sample Problem: ***Multiply.*** $2x^2(3x^2 - 4x + 1)$

Solution: $2x^2(3x^2 - 4x + 1) = 2x^2(3x^2) - 2x^2(4x) + 2x^2(1) = 6x^4 - 8x^3 + 2x^2$

Student Practice: Multiply.

1. $2x(3x + 4)$

2. $-2x^2(x^2 - 3x)$

3. $3t^2(-5t + 2)$

4. $5x^3(x^3 + 5x^2 - 6x - 2)$

• Introduction to Factoring

To **factor** an expression means to write the expression as a product of two or more factors.

Sample Problem: ***Factor each expression.*** a. 15 b. $x^2 + 3x + 2$

Solution: a. $15 = 5 \bullet 3$ b. $x^2 + 3x + 2 = (x + 2)(x + 1)$

Note that 5 and 3 are called factors of 15, while (x + 2) and (x + 1) are called factors of $x^2 + 3x + 2$.

● FACTORING OUT GCF FROM POLYNOMIALS

The **GCF (Greatest Common Factor)** is the product of the largest factor of the numerical coefficients and, if all terms have a common variable, the smallest degree of the variable. To factor polynomials, we first look for a **GCF** in all terms and use the distributive property in reverse.

Sample Problem: ***Factor out the GCF.*** $24x^5 - 30x^3$

Solution: The GCF of 24 and 30 is 6. Since both terms have an *x*, then the GCF between x^5 and x^3 is x^3 *(smallest degree).* So the GCF is $6x^3$. We can now write the expression as follows:

$24x^5 - 30x^3 = 6x^3 \bullet 4x^2 - 6x^3 \bullet 5 = 6x^3(4x^2 - 5)$ ***Check:*** $6x^3(4x^2 - 5) = 24x^5 - 30x^3$

Student Practice: Factor out the GCF of each polynomial using the distributive property. Check your answer by multiplying.

5. $15x + 25$

6. $12y - 4$

7. $50x^2 - 10x$

8. $x^3 + 9x^2$

9. $9y^2 + 6y$

10. $4x^4 - 8x^3$

Sample Problem: ***Multiply.*** $(x+5)(x-3)$

Solution:

$$(x+5)(x-3) = x(x) - x(3) + 5(x) - 5(3) = x^2 - 3x + 5x - 15 = x^2 + 2x - 15$$

Student Practice: Multiply.

1. $(x+7)(x-2)$

2. $(x-5)(x+1)$

3. $(x+5)(x-5)$

4. $(x-4)(x-4)$

• FACTORING TRINOMIALS (3 TERMS)

To factor trinomials of the form $x^2 + bx + c$,

1. Write two sets of parenthesis as such: $x^2 + bx + c = (x \qquad)(x \qquad)$
2. Place an *x* as the first term in each binomial.
3. Find two factors of *c* that sum up to the middle coefficient *b*. Write these factors as the second term of each binomial.
4. **CHECK YOUR ANSWER BY FOILING**.

It is important to note that there are two types of trinomials; those with a coefficient of 1 for the x^2, such as $x^2 + 5x + 4$; and those with a coeffiecient of something else, such as $3x^2 + 5x - 2$. Those with a coefficient of 1 for the x^2 are easier to factor and can be done as follows.

Sample Problem 1: ***Factor.*** $x^2 - 10x - 24$

Solution: $x^2 - 10x - 24 = (x \qquad)(x \qquad)$

We know the first two terms must be *x*. To find what numbers to use, we list all numbers that multiply to –24 and find the combination that adds to –10.

–24	
1	24
2	**–12**
3	8
4	6

This sum will be –10.

Will only add to –10 if we use –4 and –6, but product will then be +24.

$x^2 - 10x - 24 = (x+2)(x-12)$

Check: $(x-12)(x+2) = x^2 + 2x - 12x - 24$
$= x^2 - 10x - 24$

Sample Problem 2: ***Factor.*** $x^2 - 9x + 18$

Solution: $x^2 - 9x + 18 = (x \quad)(x \quad)$

We know the first two terms must be *x*. To find what numbers to use, we list all numbers that multiply to + 18 and find the combination that adds to –9.

+18	
1	18
2	9
–3	**–6**

This sum will be -9.

$x^2 - 9x + 18 = (x-3)(x-6)$

Check: $(x-3)(x-6) = x^2 - 6x - 3x + 18$
$= x^2 - 9x + 18$

Note: The order in which the factors are written is not important. What IS important is the sign used with each factor.

Tip: If the constant term "c" is <u>POSITIVE</u>, the numbers you are looking for must be the <u>SAME SIGN</u> and will <u>ADD</u> to the middle term (must be sign of the middle "b" term). If the constant term "c" is <u>NEGATIVE</u>, the numbers you are looking for must be <u>OPPOSITE SIGNS</u> and will <u>SUBTRACT</u> to the middle term (sign of the middle term goes to the larger of the numbers).

Student Practice: Factor each trinomial.

5. $x^2 + 8x + 12$

6. $x^2 - 3x - 18$

7. $x^2 + 25x + 24$

8. $x^2 - 13x + 36$

9. $y^2 - y - 72$

10. $x^2 - 7x - 10$

11. $x^3 + x^2 - 12x$

12. $36x - 18x^2 + 2x^3$

Sample Problem: ***Multiply.*** $(2x+5)(x-3)$

Solution:

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$(2x+5)(x-3) = 2x(x) - 2x(3) + 5(x) - 5(3) = 2x^2 - 6x + 5x - 15 = 2x^2 - x - 15$

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Student Practice: ***Multiply.***

1. $(2x+7)(x-2)$

2. $(3x-5)(4x+1)$

Factoring $ax^2 + bx + c$

Sample Problem: ***Factor.*** $5x^2 + 17x - 12$

Solution:

$5x^2 + 17x - 12 = (5x \quad 3)(x \quad 4)$

$5x^2 + 17x - 12 = (5x \quad 3)(x \quad 4)$ — $5x^2$, 12; $3x$, $20x$

$5x^2 + 17x - 12 = (5x \quad 3)(x \quad 4)$ — $5x^2$, 12; $-3x$, $+20x$, $+17x$

So, $5x^2 + 17x - 12 = (5x-3)(x+4)$ ***Check:*** $(5x-3)(x+4) = 5x^2 + 20x - 3x - 12$

$= 5x^2 + 17x - 12$

Note:** **Whenever the coefficient of x^2 ***is 1, the factors chosen for the last term can be placed in any order and must add to the middle term. However, if the coefficient of*** x^2 ***is not 1, then the factors chosen for the last term must be placed in the proper order and the inner/outer products must sum up to the middle term.***

Try: $7x^2 + 31x - 20$

Student Practice: Factor each trinomial.

3. $2x^2 - 7x + 5$

4. $3x^2 + 7x - 6$

5. $8x^2 - 18x - 5$

6. $3x^2 - 26x + 16$

7. $4x^5 - x^4 - 5x^3$

Factoring Difference of Two Squares (DOTS)

● FACTORING BINOMIALS (2 TERMS)

Notice what happens when we multiply two binomials with the same two terms, only one with addition and one with subtraction.

$(x+7)(x-7) = x^2 - 7x + 7x - 49 = x^2 - 49$

The result is a binomial that is a **difference of two perfect squares**. In other words, $x^2 - 49 = (x)^2 - (7)^2$

Tip: You can also view binomials as trinomials with a 0 middle term, such as $x^2 + 0x - 49$***, and then factor as a trinomial finding factors of -49 that add to 0.***

Any binomial that is a **difference** of **two perfect squares** can be factored using **DIFFERENCE OF TWO SQUARES**.

$$a^2 - b^2 = (a+b)(a-b)$$

Sample Problem: ***Factor.*** $x^2 - 9$

Solution: $x^2 - 9 = (x)^2 - (3)^2 = (x+3)(x-3)$

Note: Tthe two terms in the binomial MUST be perfect squares and there MUST be a difference (subtraction).

Student Practice: Factor each binomial. Remember to first check for a GCF.

1. $x^2 - 25$
2. $y^2 - 36$
3. $27a^3 - 3a$
4. $2x^2 - 32y^2$

5. $w^2 + 16$

6. $x^4 - 81$

7. $4x^3 - 9x$

8. $x^2 - 12$

Solving Quadratic Equations by Factoring

A **quadratic equation** is any equation that can be written in the standard form

$$ax^2 + bx + c = 0$$

where *a*,*b*, and *c* are real numbers and *a* > *0*. In other words, a quadratic equation is any equation with a polynomial of degree 2 (x^2 term). A quadratic equation may have up to 2 real solutions.

Sample quadratic equation: $x^2 + 4x - 21 = 0$

Principle of Zero Products

An equation $a \bullet b = 0$ is true if and only if *a = 0* or *b = 0*, or both are true. In other words, a product can be 0 only if one or both factors are 0.

Sample Problem: ***Solve for x.*** $(x-1)(x+5) = 0$

Solution. In this case, the product of $(x-1)$, and $(x+5)$ is equal to 0. This can only be true of one of these factors is equal to 0, so we determine what value of *x* makes each factor 0 by solving each equation:

$x - 1 = 0$ $\quad$ $x + 5 = 0$

$x = 1$ $\quad$ $x = -5$

Check:

$$(1-1)(1+5) = 0$$
$$(0)(6) = 0$$
$$0 = 0$$

Student Practice: Use the zero factor property to solve for x.

1. $(x+3)(x-5)=0$
2. $(2x+5)(x-9)=0$
3. $x^2-6x=0$
4. $x^2+7x-18=0$

To solve **quadratic equations by factoring**,

1. Write the equation in standard form by moving all terms to one side. Remember standard form implies *a > 0*
2. Factor the polynomial completely. Remember to first factor out the GCF.
3. Set each factor containing a variable equal to 0 and solve each equation for the variable.
4. CHECK YOUR SOLUTIONS

Sample Problem: ***Solve for x.*** $x^2-5x=14$

Solution: We must first write the equation in standard form.

$$x^2-5x=14$$
$$x^2-5x-14=0$$
$$(x-7)(x+2)=0$$
$$x-7=0 \qquad x+2=0$$
$$x=7 \qquad x=-2$$

Check:

$$(7)^2-5(7)=14$$
$$49-35=14$$
$$14=14$$

Student Practice: Solve each quadratic equation.

5. $x^2 - 4x - 21 = 0$

6. $2x^2 - 4x = 0$

7. $x^2 + 3x = 28$

8. $5x^2 = 3x + 14$

9. $x^2 = 64$

10. $-x^2 + 10x = 24$

11. $-5y + 12y^2 = 2$

12. $x(x + 4) = 12x + 20$

Applications Of Quadratic Equations

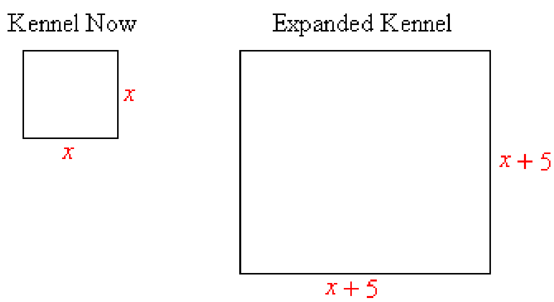
● SOLVING APPLICATION PROBLEMS

To solve word problems, try the following steps:

1. Read and understand the problem. Choose a variable to represent what you are looking for.
2. Determine an equation that can be used to solve the problem using words.
3. Then translate the words into mathematical expressions based on the variable you chose.
4. Solve the equation.
5. Check to see if the answer makes sense.

Sample Problem: A square shaped kennel will be expanded by 5 ft to increase its area. **The area of the expanded kennel will be 81 square feet**. What is the dimension of the kennel now?

Solution: Let's illustrate the scenario to help us better understand the problem. We know the kennel is a square, but we don't know the dimensions. So let's just say it is an x by x kennel.



Area of the expanded kennel is 81 sq. ft.

$$(x+5)(x+5) = 81$$

$$x^2 + 10x + 25 = 81$$

$$x^2 + 10x - 56 = 0$$

$$(x+14)(x-4) = 0$$

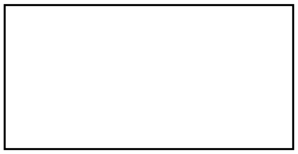
$$x = -14, x = 4$$

The original kennel was 4 by 4, and will now be 9 by 9.

Note that although x = -14 is mathematically correct, it is not a practical answer since it is not reasonable to have a kennel that measures –14 by –14 .

Student Practice: Solve each application problem.

1. The length of a rectangle is one less than two times its width. If the area is 28 square centimeters, find the length and width of the rectangle.



2. The height h of a model rocket launched from the ground into the air after t seconds can be found using the equation

$$h = 208t - 16t^2$$

a) At what time will the rocket first reach a height of 352 feet?

b) At what time will the rocket reach the ground?

3. While hovering in a helicopter 1200 feet high, a man drops his cell phone. The height *h* of the cell phone after *t* seconds is given by the formula

$$h = -16t^2 + 1200$$

a) After how many seconds will the cell phone be 800 feet high?

b) After how many seconds will the cell phone hit the ground?